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RTCC REQUIREMENTS FOR MISSIONS F AND G: EMPIRICAL EQUATIONS FOR SIMULATING THE TRANSLUNAR INJECTION MANEUVER

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Change 2 inserted —

By Jerome D. Yencharis,
Lunar Mission Analysis Branch

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MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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CHANGE SHEET

FOR

MSC INTERNAL NOTE 68-FM-154 DATED JUNE 25, 1968

RTCC REQUIREMENTS FOR MISSIONS F AND G: EMPIRICAL

EQUATIONS FOR SIMULATING THE TRANSLUNAR INJECTION MANEUVER

By Jerome D. Yencharis

Change 1

July 25, 1968

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Page 1 of 2
(with enclosures)

NOTE: A black bar in the margin indicated the area of change. After the attached enclosure, which is a replacement, has been inserted, place this CHANGE SHEET between the cover and title page and write on the cover, "CHANGE 1 inserted".

CHANGE HISTORY FOR 68-FM-154

Change no.	Date	Description
1	7/25/68	Changes to the form for the alternate mission polynomials on page 4 correct the original document.

CHANGE SHEET

FOR

MSC INTERNAL NOTE 68-FM-154 DATED JUNE 25, 1968

RTCC REQUIREMENTS FOR MISSIONS F AND G:

EMPIRICAL EQUATIONS FOR SIMULATING THE TRANSLUNAR INJECTION MANEUVER

By Jerome D. Yencharis

Change 2

October 7, 1968

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Page 1 of 3
(with enclosures)

After the attached enclosures, which are replacements, have been inserted, and the following pen-and-ink changes have been made, place this CHANGE SHEET between the cover and title page and write on the cover, "CHANGE 2 inserted".

NOTE: A black bar in the margin indicates the area of change.

1. On page 3 under independent variables in the polynomial, change the equation defining X_5 to $X_5 = R_I$, e.r.

2. On page 4 change the equation defining ΔV_m to
$\Delta V_m = (C_3 + 2\mu/R_I)^{1/2} - (\mu/R_I)^{1/2}$

3. On page 4 under independent variables for the polynomials, change the equation defining X_5 to $X_5 = R_I$

4. On page 9, the ΔV value for the coefficient a_3 should be changed to -0.17104963×10^4

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PROJECT APOLLO

RTCC REQUIREMENTS FOR MISSIONS F AND G: EMPIRICAL EQUATIONS FOR SIMULATING THE TRANSLUNAR INJECTION MANEUVER

By Jerome D. Yencharis
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June 25, 1968

MISSION PLANNING AND ANALYSIS DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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RTCC REQUIREMENTS FOR MISSIONS F AND G:

EMPIRICAL EQUATIONS FOR SIMULATING THE TRANSLUNAR INJECTION MANEUVER

By Jerome D. Yencharis

SUMMARY AND INTRODUCTION

This note gives the empirical equations to be utilized in the simulation of the translunar injection maneuver in the RTCC processor described in reference 1. These equations have been derived by TRW Systems under Project Apollo Task MSC/TRW A-77.

This document replaces reference 2 as a requirement specification document for the RTCC logic for Mission F and those following. The lunar orbit insertion (LOI) polynomials included in reference 2 are no longer necessary, since the LOI maneuver is now being simulated with an impulsive maneuver in the RTCC midcourse correction processor (ref. 3).

The following four requirements are specified by this document:

1. Polynomial forms - The polynomial forms, of which there are two, are specified within the text of this note.

2. Polynomial coefficients - Two sets of coefficients (tables I and II) are specified.

3. The criteria for the decision to use a given polynomial with its set of coefficients, and the restrictions which make up this criteria.

4. The method for using the polynomials .

The empirical simulation of the TLI burn gives the burn characteristics of an optimum thrusting (minimum ΔV) maneuver which takes the vehicle from a given set of initial conditions to a specified set of end conditions. This technique represents a calculus of variation solution to the problem (ref. 4). The set of initial conditions specifies an earth parking orbit and vehicle characteristics (e.g., thrust, weight). The set of end conditions specifies a hypersurface (fig. 1).

(The hypersurface is described in references 4 and 5 and will not be discussed.)

POLYNOMIAL FORMS AND COEFFICIENTS

Two basic polynomials are required to simulate the translunar injection maneuver in the RTCC processor described in reference 1. The criteria for deciding which form to use is specified in the next section, "Criteria for Use of a Given Polynomial Form."

The two polynomial forms are the "Nominal Mission Polynomial" and the "alternate mission polynomial." This terminology is used for identification purposes only. Both sets of polynomials accept the same input parameters and remit the same output parameters.

The independent variables for both polynomials are

1. C_3 - twice vis-viva energy desired at cutoff, $(\text{e.r./hr})^2$.

2. σ - central angle between the unit target vector $(\overline{T})$ and the perigee vector on any trajectory corresponding to a given hypersurface, radians.

3. δ - the declination of the target vector with respect to the parking orbit plane; positive measured toward the angular momentum vector of the earth parking orbit, radians.

4. F/W - the thrust-to-weight ratio at S-IVB ignition, lbf/lbm.

5. R_I - the magnitude of the earth-centered position vector of the S-IVB at ignition, e.r.

The output parameters of the polynomial, illustrated in figure 2, are

1. ΔV - the characteristic velocity for the maneuver, e.r./hr.

2. α - angle measured in the parking orbit plane from the S-IVB ignition point to the projection of the target vector in the parking orbit plane; positive in the direction of motion, radians.

3. β - angle measured in the parking orbit plane from the S-IVB ignition point to the intersection of the parking orbit plane and the plane of the cutoff ellipse; positive in the direction of motion, radians.

4. η - true anomaly at translunar injection cutoff, radians.

5. R_p - radius of perigee on the translunar ellipse at cutoff, e.r.

The five sets of coefficients for each polynomial are thus used to determine the five output parameters.

The nominal mission polynomial coefficients are listed in table I. The alternate mission polynomial coefficients are listed in table II.

The form for the nominal mission polynomial is derived from reference 6. (See reference 7 also.)

$$\begin{aligned} Y_I &= a_o + a_1X_1 + a_2X_2 + a_3X_2^2 + a_4X_2^3 + a_5X_2^4 + a_6X_3 + a_7X_3^2 + a_8X_1X_2 \\ &+ a_9X_2X_3 + a_{10}X_2^2X_3 + a_{11}X_2^3X_3 + a_{12}X_4 + a_{13}X_1X_4 + a_{14}X_2X_4 \\ &+ a_{15}X_2^2X_4 + a_{16}X_3X_4 + a_{17}X_3^2X_4 + a_{18}X_2^2X_3X_4 + a_{19}X_5 + a_{20}X_1X_3 \\ &+ a_{21}X_1X_2^2 + a_{22}X_1X_2^2X_3 \end{aligned}$$

where Y_I $(I = 1,2,3,4,5) = \alpha,\ \beta,\ \eta + \alpha,\ R_p,\ \Delta V$.

For β, the additional term to the polynomial is needed:

$$\beta = Y_\beta + \frac{X_4^2\ (a_{23} + a_{24}X_1 + a_{25}X_4^2)}{(X_3 + 0.148)^2 + 4(X_2 + 0.0027)^2}$$

where Y_β is the 23 term polynomial for β.

The independent variables in the polynomial are

$X_1 = \Delta V_m - 1.75$, e.r./hr

$X_2 = \Delta i^2 - 0.0027$, rad

$X_3 = \sigma - 0.148$, rad

$X_4 = 1.0/(F/W)$

$X_5 = R_i$, e.r.

where

$$\Delta V_m = (C_3 + 2\mu/R_i)^{1/2} - (\mu/R_i)^{1/2}$$

$$\Delta i = \sin^{1}\left[\frac{\sin|\delta|}{\sin(\sigma + 0.314)}\right]$$

Table I gives the coefficients for determining ΔV, α, β, $\eta + \alpha$, and R_p. η is determined by evaluating the polynominals for α and for $\eta + \alpha$ and subtracting the result of the former from the results for the latter.

The form for the alternate mission polynomials is from reference 8. The polynominal form is

$$\begin{aligned} Y_I = {} & a_o + a_1X_1 + a_2X_1^2 + a_3X_2 + a_4X_2^2 + a_5X_2^3 + a_6X_1X_2 + a_7X_1^2X_2^2 \\ & + a_8X_1^2X_2 + a_9X_1X_2^2 + a_{10}X_1X_2^3 + a_{11}X_3 + a_{12}X_3^2 + a_{13}X_1X_3 + a_{14}X_2X_3 \\ & + a_{15}X_1X_2X_3 + a_{16}X_2^2X_3 + a_{17}X_1X_2^3X_3 + a_{18}X_1^2X_2^3X_3 + a_{19}X_4 + a_{20}X_1X_4 \\ & + a_{21}X_1^2X_4 + a_{22}X_1X_2X_3X_4 + a_{23}X_1^2X_2X_3X_4 + a_{24}X_1^2X_2^2X_3X_4 + a_{25}X_1^2X_2X_3^2X_4 \\ & + a_{26}X_5 + a_{27}X_1X_5 \end{aligned}$$

For β, additional terms are required:

$$\beta = Y_\beta + \frac{X_1X_4 \quad (a_{28} + a_{29}X_1 + a_{30}X_1X_4 + a_{31}X_3^2)}{(X_3 + 0.40)^2 + 3X_2^2}$$

(Y_I and Y_β are defined as above for the nominal mission polynomials.)

The independent variables for the polynomials are:

$$X_1 = \Delta V_m$$

$$X_2 = \frac{(\Delta i)^2}{(|\Delta i| + 0.006X_1^2)}$$

$$X_3 = \sigma - 0.40$$

$$X_4 = 1.0/(F/W)$$

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$$X_5 = R_i$$

where ΔV_m is defined as for the nominal mission polynomials, and

$$\Delta i = \sin^{-1}\left[\frac{\sin \delta}{\sin (\sigma + 0.13)}\right]$$

Table II gives the coefficients for determining the values of α, β, $\eta + \alpha$, ΔV and R_p. η is found by the same procedure that was outlined above for the nominal mission polynomials.

CRITERIA FOR USE OF A GIVEN POLYNOMIAL FORM

The range of values for each independent parameter for which each polynomial form is valid is shown in table III. The accuracy of the output parameters is given in table IV. To maintain this accuracy, it is necessary that the following restrictions be adhered to.

1. The criteria for using one polynomial form or the other is the value of C_3 which is specified. C_3 is related to the semimajor axis, a, by the following equation -

$$C_3 = -\frac{\mu}{a}$$

To use the alternate mission polynomial, C_3 must be in the range from $-45\ km^2/sec^2$ to $-5\ km^2/sec^2$, or, in terms of the semimajor axis, a must be in the range from 4793 n. mi. to 43 046 n. mi. To use the nominal mission polynomial, C_3 must be in the range from $-5\ km^2/sec^2$ to $-0.5\ km^2/sec^2$, or the semimajor axis a must be in the range from 43 046 n. mi. to 430 460 n. mi.

2. The lowest apogee altitude for which the alternate mission polynomial is valid is 2600 n. mi.

3. The use of these polynomials in an iterative processor (such as that outlined in ref. 1) should adhere to the additional rule that, if the independent parameter δ is input to the polynomial and has a value outside of the prescribed limits, the following process be followed:

If $\delta > 2^\circ$, set $\delta_o = 2^\circ$ and $\Delta\delta = \delta - 2^\circ$

If $\delta < -2^\circ$, set $\delta_o = -2^\circ$ and $\Delta\delta = -2^\circ - \delta$

Then let α_o, β_o, $(\eta + \alpha)_o$, R_{p_o}, ΔV_o denote the values of the polynomials obtained by setting δ equal to δ_o and leaving all other inputs unchanged. These values are then adjusted by the following equations:

$$\alpha = \alpha_o - 4.66\ \Delta\delta$$

$$\beta = \beta_o - 2.15\ \Delta\delta$$

$$\eta + \alpha = (\eta + \alpha)_o + 0.923\ \Delta\delta$$

$$R_p = R_{p_o} - 0.442\ \Delta\delta$$

$$\Delta V = \Delta V_o + 6.33\ \Delta\delta$$

where α, β, η and $\Delta\delta$ are in radians, R_p is earth radii, and ΔV is in e.r./hr.

4. Again, if the polynomials are used in an iterative process and the first guess on C_3 is near the "common" point for both polynomials ($-5.0\ km^2/sec^2$), only one polynomial form is to be used in the process. If the solution appears to require a value for C_3 corresponding to the other polynomial form, the iterative process should be reinitialized, using the appropriate polynomial form.

The above restrictions are to be followed in the implementation of the polynomials in the RTCC processor described in reference 1.

METHODS FOR USING THE POLYNOMIALS

The inputs to TLI simulation are C_3, σ, δ, F/W, and R_I, which have been defined, and V_I, W_I, F_I, F and T_{MRS}, which are defined below.

The output of the TLI simulation is usually the state vector at TLI cutoff. The state is computed, using the output of the TLI polynomials . A method for doing this is presented in the appendix.

To use the polynomials in the RTCC, it is necessary that relevent off-nominal situations be accounted for. Therefore, the following procedure (from ref. 6) should be implemented to account for mixture ratio shifts during the TLI burn and non-circular conditions in the orbit at S-IVB ignition.

1. Compute ΔV_I, ΔT_{B1} and $\Delta\phi_{B1}$.

$$\Delta V_I = (\mu/R_i)^{1/2} - V_I$$

where V_I is the actual velocity magnitude at ignition.

$$\Delta T_{B1} = \frac{\Delta V_I}{(F/W_I)\ g_o}$$

where W_I is the actual weight at ignition and g_o is the gravitational acceleration at the earth's surface.

$$\dot{\phi}_I = V_I/R_I$$

$$\Delta\phi_{B1} = \dot{\phi}_I\ \Delta T_{B1}$$

2. Compute ΔT_{B2} and $\Delta\phi_{B2}$.

$$\Delta T_{B2} = \left[1 - \frac{F_I}{F}\right]\ T_{MRS}$$

where F_I is the predicted thrust magnitude from ignition to T_{MRS}

F is the predicted thrust magnitude from T_{MRS} to cutoff

T_{MRS} is the estimated time of the mixture ratio shift, measured from ignition.

$$\Delta\phi_{B2} = \dot{\phi}_I\ \Delta T_{B2}$$

3. Adjust the weight, W, used in the independent variable F/W.

$$W = W_I - \dot{W}\ \Delta T_{B1}$$

where $\dot{W}$ is the nominal weight flow rate and is taken to be positive.

4. The polynomials are now evaluated in the normal manner, noting that in the independent parameter F/W, F is taken to be the nominal thrust value (or, in the case of a mixture ratio shift, the predicted magnitude of the thrust after the mixture ratio shift), and W is adjusted as in step 3.

5. The output of the polynomials is adjusted in the following manner:

$$\alpha = P(\alpha) + \Delta\phi_{B1} + \Delta\phi_{B2}$$

$$\beta = P(\beta) + 3/4\ \Delta\phi_{B1} + \Delta\phi_{B2}$$

$$\eta = P(\eta+\alpha) - P(\alpha)$$

$$R_P = P(R_P)$$

$$\Delta V = P(\Delta V) + \Delta V_I$$

where $P(\alpha)$ is the polynomial for α, etc.

6. To get the total burn time (T_B) for the maneuver

$$T_B = \left[\frac{W_I}{\dot{W}}\right]\left\{1 - e^{-\left[(\Delta V/g_o)(\dot{W}/_F)\right]}\right\} + \Delta T_{B2} \quad .$$

The above procedure is to be implemented in the RTCC for the TLI simulations.

TABLE I. - COEFFICIENTS FOR THE NOMINAL MISSION

TLI BURN SIMULATION POLYNOMIALS

Coefficients	α, rad	β, rad	$\eta + \alpha$, rad	R_p, e.r.	ΔV, e.r./hr
a_0	$0.61967804 \times 10^{-1}$	0.51541772	0.48329414	$0.46986962 \times 10^{-2}$	0.18679213×10^{1}
a_1	$0.86219648 \times 10^{-2}$	-0.15528032	$0.18759385 \times 10^{-2}$	$-0.44724715 \times 10^{-2}$	0.98320266
a_2	$-0.24371820 \times 10^{2}$	0.27185659×10^{2}	0.14031932×10^{1}	$-0.17477015 \times 10^{1}$	0.25028715×10^{2}
a_3	0.41004848×10^{4}	0.18763984×10^{3}	$-0.13933485 \times 10^{3}$	0.16880382×10^{2}	0.17104963×10^{4}
a_4	$-0.99229657 \times 10^{6}$	$-0.92712145 \times 10^{6}$	0.40515931×10^{5}	0.14554490×10^{5}	0.37348295×10^{6}
a_5	0.14267564×10^{9}	0.21114994×10^{9}	$-0.48676865 \times 10^{7}$	$-0.27167564 \times 10^{7}$	$-0.51521225 \times 10^{8}$
a_6	-0.54688962	-0.56424215	$-0.10155877 \times 10^{1}$	$0.25758493 \times 10^{-1}$	-0.21640574
a_7	-0.78766288	0.95105384×10^{1}	$0.83266987 \times 10^{-1}$	$-0.77608981 \times 10^{-1}$	0.41744541
a_8	0.10261969×10^{2}	$-0.15294910 \times 10^{2}$	$-0.28021958 \times 10^{1}$	0.57075666	$-0.65859807 \times 10^{1}$
a_9	0.52445599×10^{1}	0.33896643×10^{2}	0.21207686	0.36716041×10^{1}	$-0.57578939 \times 10^{1}$
a_{10}	$-0.15527983 \times 10^{5}$	$-0.26903240 \times 10^{5}$	0.98814614×10^{3}	$-0.96377142 \times 10^{3}$	0.99505664×10^{4}
a_{11}	0.51931839×10^{7}	0.12131396×10^{8}	$-0.17699125 \times 10^{6}$	$-0.56658473 \times 10^{5}$	$-0.20041678 \times 10^{7}$
a_{12}	0.18005069	0.25371175	0.30964851	$0.61201761 \times 10^{-2}$	$-0.76178399 \quad 10^{-2}$
a_{13}	$0.97069489 \times 10^{-1}$	0.22036833	0.13152495	$0.72885987 \times 10^{-2}$	$-0.14737246 \times 10^{-2}$
a_{14}	0.61442230×10^{1}	$-0.22601576 \times 10^{2}$	0.92808415	0.55059398	$-0.43988981 \times 10^{1}$
a_{15}	$-0.87765197 \times 10^{3}$	0.14378586×10^{4}	$-0.32524984 \times 10^{2}$	0.44179572×10^{1}	0.20554193×10^{3}
a_{16}	-0.16502383	0.31264540	$0.44675108 \times 10^{-2}$	$-0.10348462 \times 10^{-1}$	$0.74293949 \times 10^{-1}$
a_{17}	0.63224468	$-0.64046690 \times 10^{1}$	$-0.59053312 \times 10^{-1}$	$0.49107017 \times 10^{-1}$	-0.29415058
a_{18}	0.81844028×10^{3}	$-0.39254760 \times 10^{4}$	$-0.10061669 \times 10^{3}$	0.44830843×10^{3}	$-0.16188456 \times 10^{4}$
a_{19}	-0.33505204	-0.57987931	-0.60405621	0.98814646	$-0.30007513 \times 10^{-1}$
a_{20}	$-0.92426341 \times 10^{-1}$	-0.22905910	$0.96317404 \times 10^{-2}$	$-0.73420686 \times 10^{-2}$	$0.28463657 \times 10^{-1}$

TABLE I. - COEFFICIENTS FOR THE NOMINAL MISSION

TLI BURN SIMULATION POLYNOMIAL - Concluded

Coefficients	α, rad	β, rad	$\eta + \alpha$, rad	R_p, e.r.	ΔV, e.r./hr
a_{21}	-0.18131458×10^4	0.12621438×10^4	0.18026336×10^3	-0.27739134×10^2	0.79392771×10^3
a_{22}	-0.39193696×10^4	0.70516077×10^4	0.81684373×10^2	0.29172916×10^3	0.12182700×10^4
a_{23}	--	$-0.76940409 \times 10^{-4}$	--	--	--
a_{24}	--	$0.64393915 \times 10^{-4}$	--	--	--
a_{25}	--	$0.48483478 \times 10^{-4}$	--	--	--

TABLE II.- COEFFICIENTS FOR ALTERNATE MISSION TLI SIMULATION POLYNOMIALS

Coefficients	α(rad)	β(rad)	$\eta + \alpha$(rad)	R_p(er)	ΔV(er/hr)
a_0	-0.46115370	-0.30106979	-0.34400710	$0.45577371 \times 10^{-2}$	$0.44538134 \times 10^{-1}$
a_1	0.22773466	0.71115079	0.35035981	0.0	0.98663545
a_2	$0.19555311 \times 10^{-2}$	$-0.68187747 \times 10^{-1}$	$-0.69479865 \times 10^{-2}$	$-0.50707834 \times 10^{-3}$	$-0.56569620 \times 10^{-2}$
a_3	$-0.34314288 \times 10^{1}$	0.30524303×10^{1}	0.33711079	$0.13575870 \times 10^{-1}$	0.65162585
a_4	0.10972731×10^{2}	$-0.77950375 \times 10^{2}$	0.60217170×10^{1}	$-0.18127728 \times 10^{1}$	0.27300140×10^{2}
a_5	$-0.61556301 \times 10^{3}$	0.86691181×10^{2}	$-0.47852738 \times 10^{1}$	0.73544148×10^{1}	$-0.56837040 \times 10^{1}$
a_6	0.68371262×10^{1}	$-0.43251893 \times 10^{1}$	-0.57183248	$0.69169491 \times 10^{-1}$	$-0.12413913 \times 10^{1}$
a_7	0.27942824×10^{2}	$-0.43827950 \times 10^{2}$	-0.67084710	0.21581974	$-0.36245037 \times 10^{1}$
a_8	$-0.21180981 \times 10^{1}$	0.19927943×10^{1}	0.19946202	$-0.30721717 \times 10^{-1}$	0.41329476
a_9	$-0.11490251 \times 10^{3}$	0.13794269×10^{3}	0.16108986×10^{1}	0.12382241	0.89385858×10^{1}
a_{10}	0.14027177×10^{4}	$-0.19130249 \times 10^{3}$	$-0.32165452 \times 10^{2}$	0.26043273×10^{1}	$-0.19328518 \times 10^{3}$
a_{11}	$-0.10215248 \times 10^{1}$	$-0.81325347 \times 10^{-1}$	$-0.10019443 \times 10^{1}$	$-0.15620168 \times 10^{-2}$	$0.38846744 \times 10^{-2}$
a_{12}	-0.11176447	-0.19995333	$-0.29683074 \times 10^{-3}$	$-0.30462187 \times 10^{-2}$	$0.15599869 \times 10^{-1}$
a_{13}	$-0.28441985 \times 10^{-1}$	-0.10653570	$0.17184573 \times 10^{-2}$	$-0.11997834 \times 10^{-3}$	$0.37744704 \times 10^{-2}$
a_{14}	0.15194679×10^{2}	0.19806250	$0.78000508 \times 10^{-1}$	0.14836169	$-0.16105487 \times 10^{1}$

TABLE II.- COEFFICIENTS FOR ALTERNATE MISSION TLI SIMULATION POLYNOMIALS - Continued

Coefficients	α(rad)	β(rad)	$\eta + \alpha$(rad)	R_p(er)	ΔV(er/hr)
a_{15}	0.17363458×10^{1}	0.12671075×10^{2}	$-0.52481997 \times 10^{-1}$	0.13979050	$-0.21292248 \times 10^{1}$
a_{16}	$-0.29872899 \times 10^{3}$	$-0.12117350 \times 10^{3}$	0.13599529×10^{2}	$-0.19255400 \times 10^{1}$	0.75144900×10^{2}
a_{17}	0.20685458×10^{4}	0.10037573×10^{4}	$-0.42908334 \times 10^{2}$	0.12275655×10^{2}	$-0.40806938 \times 10^{3}$
a_{18}	0.23445758×10^{3}	$-0.64043756 \times 10^{3}$	$-0.28623014 \times 10^{2}$	0.32929342	$-0.13406229 \times 10^{1}$
a_{19}	$-0.16844313 \times 10^{-2}$	$0.19854365 \times 10^{-1}$	$0.11456195 \times 10^{-1}$	$-0.35659736 \times 10^{-3}$	$0.35304466 \times 10^{-2}$
a_{20}	0.11073411	$0.54047018 \times 10^{-1}$	0.19812576	$0.59006873 \times 10^{-3}$	$-0.85815036 \times 10^{-2}$
a_{21}	$-0.12723911 \times 10^{-1}$	$0.34129362 \times 10^{-1}$	$-0.16722383 \times 10^{-1}$	$0.12711563 \times 10^{-2}$	$0.61631299 \times 10^{-2}$
a_{22}	-0.97939990	-0.19836786	-0.75046623	$-0.11267094 \times 10^{-1}$	$0.60883411 \times 10^{-1}$
a_{23}	0.46225223	0.46879377	0.29063760	$-0.23112987 \times 10^{-1}$	0.12879841
a_{24}	0.24985093×10^{2}	0.41743162×10^{2}	0.63805218	0.45292900	$-0.58643257 \times 10^{1}$
a_{25}	0.73621917×10^{1}	0.14016618×10^{2}	-0.20020239	0.12662005	$-0.19083570 \times 10^{1}$
a_{26}	$0.67955599 \times 10^{-1}$	0.23463883	$-0.70863131 \times 10^{-1}$	0.99587755	$-0.48126707 \times 10^{-1}$
a_{27}	-0.23841464	-0.59041779	-0.31706871	$-0.11403328 \times 10^{-2}$	$0.23212768 \times 10^{-1}$
a_{28}		$0.31853394 \times 10^{-3}$			
a_{29}		$-0.63648369 \times 10^{-4}$			

TABLE II.- COEFFICIENTS FOR ALTERNATE MISSION TLI SIMULATION POLYNOMIALS - Concluded

Coefficients	α(rad)	β(rad)	$\eta + \alpha$(rad)	R_p(er)	ΔV(er/hr)
a_{30}		$0.13602553 \times 10^{-3}$			
a_{31}		$-0.26753068 \times 10^{-2}$			

TABLE III.- RANGE OF INDEPENDENT VARIABLES

FOR THE TLI POLYNOMIALS

Parameter	Nominal Mission	Alternate Mission
C_3, km^2/sec^2 [a]	$-5.0 \leq C_3 \leq -0.5$	$-45.0 \leq C_3 \leq -5.0$
σ, deg	$2.0 \leq \sigma \leq 15.0$	$2.0 \leq \sigma \leq 15.0$
δ, deg	$0.0 \leq \lvert\delta\rvert \leq 2.0$	$0.0 \leq \lvert\delta\rvert \leq 1.0$
F/w	$0.6298 \leq F/W \leq 0.8048$	$0.6298 \leq F/W \leq 0.8048$
R_I, km	$6508.5077 \leq R_i \leq 6643.5077$	$6508.5077 \leq R_i \leq 6643.5077$

[a]The ranges on semimajor axis (a) associated with the values of C_3 are:

Nominal Mission: 43 046 n. mi. $\leq a \leq$ 430 046 n. mi.

Alternate Mission: 4793 n. mi. $\leq a \leq$ 43 046 n. mi.

The approximate apogee altitudes associated with the above semimajor axes are (assuming 100 n. mi. perigee altitude):

Nominal Mission 77 000 n. mi. $\leq h_a \leq$ 835 000 n. mi.

Alternate Mission: 2600 n. mi. $\leq h_a \leq$ 77 000 n. mi.

TABLE IV. - ACCURACY OF THE TLI POLYNOMIALS

Parameter	RMS Residual	Maximum Residual
(a) Nominal Mission		
α, deg	0.112	0.590
β, deg	0.475	3.270
$\eta + \alpha$, deg	0.013	.048
R_p, n. mi.	0.365	1.143
ΔV, fps	3.890	16.624
(b) Alternate Mission		
α, deg	0.114	0.450
β, deg	0.222	0.973
$\eta + \alpha$, deg	0.016	0.057
R_p, n. mi.	0.190	1.120
ΔV, fps	2.080	7.580

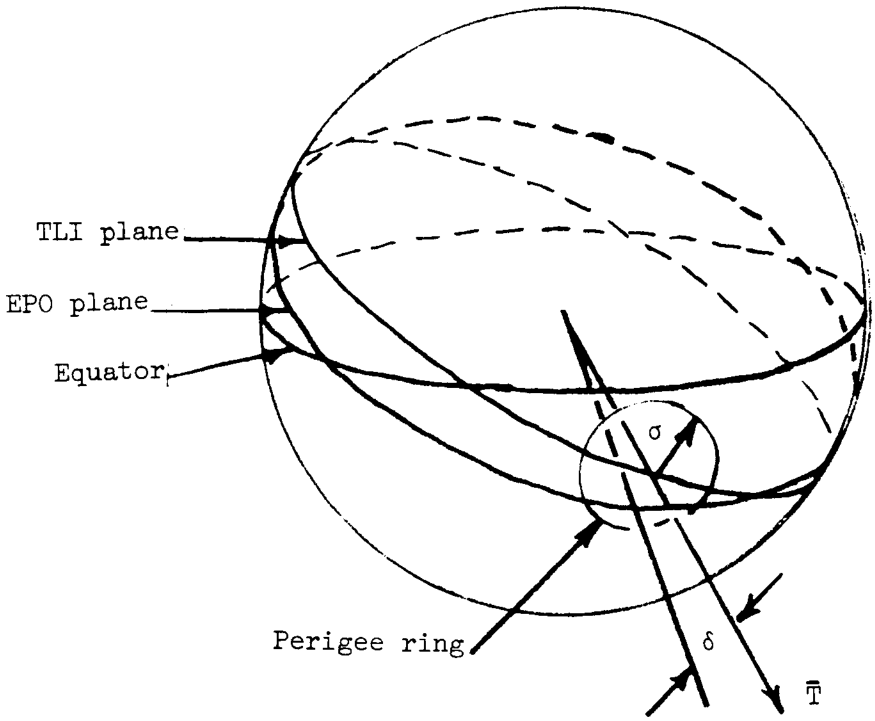

Figure 1.- Geometry of the hypersurface (ref. 4).

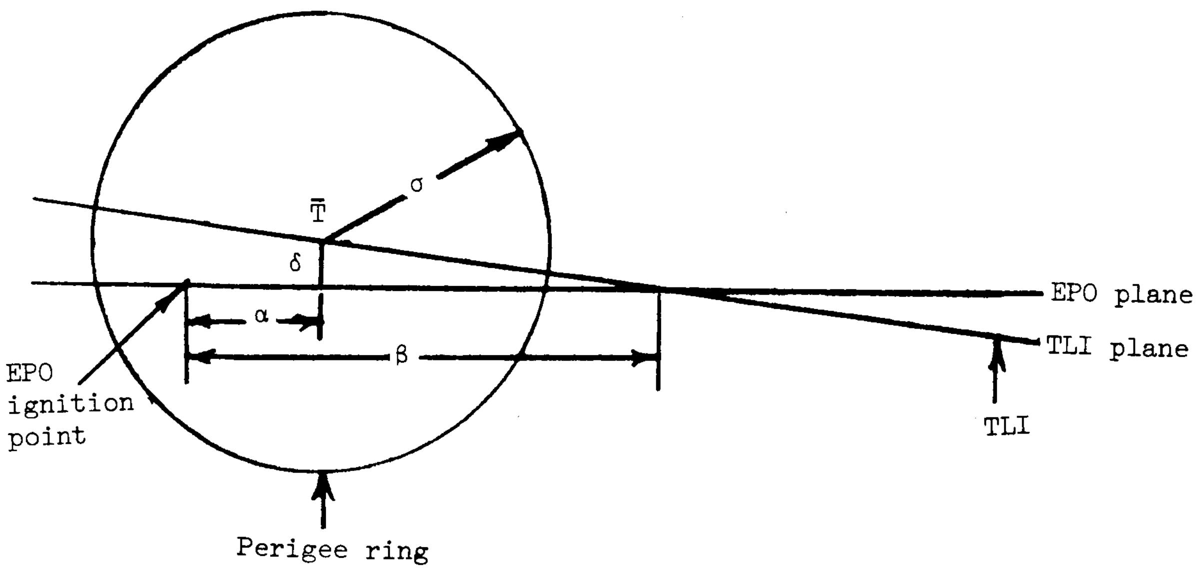

Figure 2.- Geometry associated with the empirical simulation of translunar injection (ref. 4).

APPENDIX

COMPUTING THE STATE VECTOR AT TLI CUTOFF

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COMPUTING THE STATE VECTOR AT TLI CUTOFF

Reference 8 presents a method to compute the cutoff state vector for the TLI maneuver, given the ignition state and C_3, σ, δ, R_I, F/W, and the five parameters computed by the empirical simulation. This method is repeated here, merely as information.

The unit target vector, $\bar{T}$, for the hypersurface is calculated using the ignition state $(\bar{R}_I, \bar{V}_I)$ and α and δ.

$$\hat{R}_I = \frac{\bar{R}_I}{|\bar{R}_I|}$$

$$\hat{N}_I = \frac{(\bar{R}_I \times \bar{V}_I)}{|\bar{R}_I \times \bar{V}_I|}$$

$$\bar{T} = \hat{R}_I \cos\delta \cos\alpha + \hat{N}_I \times \hat{R}_I \cos\delta \sin\alpha + \hat{N}_I \sin\delta$$

Next, the unit normal vector to the TLI plane $(\hat{N}_c)$ is calculated:

$$\hat{S} = \hat{R}_I \cos\beta + \hat{N}_I \times \hat{R}_I \sin\beta$$

$$\hat{N}_c = \frac{\bar{T} \times \hat{S}}{|\bar{T} \times \hat{S}|}$$

Next, angular momentum, C_1, semilatus rectum, p, and eccentricity, e, are computed:

$$C_1 = R_p \left(\frac{2\mu}{R_p} + C_3\right)^{1/2}$$

$$p = \frac{C_1^2}{\mu}$$

$$e = \left(1 + \frac{C_3}{\mu} p\right)^{1/2}$$

where μ = gravitational constant

The magnitudes of the cutoff position, R_c, cutoff velocity, V_c, and cutoff flight-path angle, γ_c, are computed.

$$R_c = \frac{p}{1 + e \cos \eta}$$

$$V_c = \left(\frac{\mu}{C_1}\right) (1 + 2e \cos \eta + e^2)^{1/2}$$

$$\gamma_c = \tan^{-1} \left[\frac{e \sin \eta}{1 + e \cos \eta}\right]$$

The position and velocity vectors at cutoff then are

$$\bar{R}_c = R_c \left[\bar{T} \cos (\sigma + \eta) + \hat{N}_c \times \bar{T} \sin (\sigma + \eta)\right]$$

$$\bar{V}_c = V_c \left[-\bar{T} \sin (\sigma + \eta - \gamma_c) + \hat{N}_c \times \bar{T} \cos (\sigma + \eta - \gamma_c)\right]$$

The time at the cutoff state is equal to the time at ignition, t_I, plus the time of the TLI burn, T_β, defined in the previous section.

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